**Chapter 5**

**Sequences and Series**

**5.4 Comparison Tests**

**Section Exercises**

**Use the comparison test to determine whether the following series converge.**

195.  where 

Answer: Converges by comparison with 

197. 

Answer: Diverges by comparison with harmonic series, since 

199. 

Answer: Converges by comparison with *p*-series, 

201. 

Answer: so converges by comparison with *p*-series*,* 

203. 

Answer:  so converges by comparison with *p*-series,

205. 

Answer: Since  series converges by comparison with *p*-seriesfor 

**Use the limit comparison test to determine whether each of the following series converges or diverges.**

207. 

Answer: Converges by limit comparison with *p*-seriesfor 

209. 

Answer: Converges by limit comparison with *p*-series,

211. 

Answer: Converges by limit comparison with 

213. 

Answer: Converges by limit comparison with 

215. 

Answer: Diverges by limit comparison with harmonic series.

217. 

Answer: Converges by limit comparison with *p*-series, 

219. 

Answer: Converges by limit comparison with *p*-series, 

221.  (*Hint:* so )

Answer: Diverges by limit comparison with 

223. Does  converge ifis large enough? If so, for which 

Answer: Converges for  by comparison with a  series for slightly smaller

225. For which  does the series  converge?

Answer: Converges for all 

227. For which  does the series  converge?

Answer: Converges for all  If  then  say, once  and then the series converges by limit comparison with a geometric series with ratio 

229. Does  converge or diverge? Explain.

Answer: The numerator is equal to  when  is odd and  when  is even, so the series can be rewritten  which diverges by limit comparison with the harmonic series.

231. Suppose that  and  and that  and  converge. Prove that  converges and 

Answer: or  so convergence follows from comparison of  with  Since the partial sums on the left are bounded by those on the right, the inequality holds for the infinite series.

233. Does  converge? (*Hint:* Use  to compare to a **)

Answer:  If is sufficiently large, then  so  and the series converges by comparison to a **

235. Show that if  and  converges, then  converges. If  converges, does  necessarily converge?

Answer: so  for large  Convergence follows from limit comparison.  converges, but  does not, so the fact that  converges does not imply that  converges.

237. Suppose that  for all  and that  diverges. Suppose that  is an arbitrary sequence of zeros and ones with infinitely many terms equal to one. Does  necessarily diverge?

Answer: No.  diverges. Let  unless  for some  Then  converges.

239. Show that if  and  converges, then  converges.

Answer:  so the result follows from the comparison test.

241. Let  be an infinite sequence of zeros and ones. What is the largest possible value of 

Answer: By the comparison test, 

243. Explain why, if  then  cannot be written 

Answer: If  then, by comparison, 

245. [**T]** Robert wants to know his body mass to arbitrary precision. He has a big balancing scale that works perfectly, an unlimited collection of  weights, and nine each of   and so on weights. Assuming the scale is big enough, can he do this? What does this have to do with infinite series?

Answer: Yes. Keep adding  weights until the balance tips to the side with the weights. If it balances perfectly, with Robert standing on the other side, stop. Otherwise, remove one of the  weights, and add  weights one at a time. If it balances after adding some of these, stop. Otherwise if it tips to the weights, remove the last  weight. Start adding  weights. If it balances, stop. If it tips to the side with the weights, remove the last  weight that was added. Continue in this way for the  weights, and so on. After a finite number of steps, one has a finite series of the form  where  is the number of full kg weights and  is the number of  weights that were added. If at some state this series is Robert’s exact weight, the process will stop. Otherwise it represents the  partial sum of an infinite series that gives Robert’s exact weight, and the error of this sum is at most 

247. In view of the previous exercise, it may be surprising that a subseries of the harmonic series in which about one in every five terms is deleted might converge. A *depleted harmonic series* is a series obtained from  by removing any term  if a given digit, say  appears in the decimal expansion of  Argue that this depleted harmonic series converges by answering the following questions.

1. How many whole numbers  have  digits?
2. How many  whole numbers  do not contain  as one or more of their digits?
3. What is the smallest  number 
4. Explain why the deleted harmonic series is bounded by 
5. Show that  converges.

Answer: a.  b.  c.  d. Group the terms in the deleted harmonic series together by number of digits.  bounds the number of terms, and each term is at most  e.  One can actually use comparison to estimate the value too smaller than  The actual value is smaller than 

249. Suppose that a sequence of numbers  has the property that  and  where  Can you determine whether  converges? (*Hint:*   etc. Look at  and use  

Answer: Continuing the hint gives  Then  Since  is bounded by a constant times  when  one has  which converges by comparison to the *p*-series for 

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